Uygulama

30 Mart 2022 Çarşamba 10:39

$A_{n\times n}$, $Y' = AY$ → sistem;

Lineer bağımsız n - tane çözümü $y_1(t), y_2(t), \dots, y_n(t)$ olsun.

Homojen denklemin çözümü,

$$y_h(t) = c_1 y_1(t) + c_2 y_2(t) + \dots + c_n y_n(t)$$

$$= c_1 \begin{bmatrix} y_{11}(t) \\ y_{21}(t) \\ \vdots \\ y_{n1}(t) \end{bmatrix} + c_2 \begin{bmatrix} y_{12}(t) \\ y_{22}(t) \\ \vdots \\ y_{n2}(t) \end{bmatrix} + \dots + c_n \begin{bmatrix} y_{1n}(t) \\ y_{2n}(t) \\ \vdots \\ y_{nn}(t) \end{bmatrix} = \begin{bmatrix} c_1 y_{11}(t) + c_2 y_{12}(t) + \dots + c_n y_{1n}(t) \\ \vdots \\ c_1 y_{n1}(t) + c_2 y_{n2}(t) + \dots + c_n y_{nn}(t) \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} y_{11}(t) & y_{12}(t) & \dots & y_{1n}(t) \\ \vdots & \vdots & & \\ y_{n1}(t) & y_{n2}(t) & \dots & y_{nn}(t) \end{bmatrix}}_{\Phi(t)} \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix}$$

$\Phi(t) = [\,y_1(t) \;\; y_2(t) \;\; \dots \;\; y_n(t)\,]$

$\Phi(t)$ → Temel Matris!

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$$y' = \begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix} y + \begin{bmatrix} e^{2t} \\ 10t \end{bmatrix}$$ B.D.P nin çözümünü bulunuz.

$$y(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}, \quad |A - \lambda I| = \begin{vmatrix} 4-\lambda & 2 \\ 3 & -1-\lambda \end{vmatrix} = -(4-\lambda)(1+\lambda) - 6 = -(4 + \overbrace{3\lambda}^{} - \lambda - \lambda^2) - 6$$

$$= \lambda^2 - 3\lambda - 10 = 0 \quad (-5)(2)$$

$$\lambda = -2, \quad \lambda = 5$$

- $\lambda = -2$ için

$$\begin{bmatrix} 6 & 2 \\ 3 & 1 \end{bmatrix} \underbrace{\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}}_{v} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$6v_1 + 2v_2 = 0$

$3v_1 + v_2 = 0$

$v_2 = -3v_1$

$$v = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

- $\lambda = 5$

$$\begin{bmatrix} -1 & 2 \\ 3 & -6 \end{bmatrix} \underbrace{\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}}_{u} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$-u_1 + 2u_2 = 0$

$3u_1 - 6u_2 = 0$

$u_1 = 2u_2$

$$u = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

- $(5, \begin{bmatrix} 2 \\ 1 \end{bmatrix}) \to y_1(t) = e^{5t} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2e^{5t} \\ e^{5t} \end{bmatrix}$

- $(-2, \begin{bmatrix} 1 \\ -3 \end{bmatrix}) \to y_2(t) = e^{-2t} \begin{bmatrix} 1 \\ -3 \end{bmatrix} = \begin{bmatrix} e^{-2t} \\ -3e^{-2t} \end{bmatrix}$

$W(y_1, y_2) \neq 0$

$$\Phi(t) = [y_1, y_2] = \begin{bmatrix} 2e^{5t} & e^{-2t} \\ e^{5t} & -3e^{-2t} \end{bmatrix}$$

$$y_h(t) = \Phi(t)\Phi^{-1}(t_0)y_0$$

$$\Phi(0) = \begin{bmatrix} 2 & 1 \\ 1 & -3 \end{bmatrix} \Rightarrow \Phi^{-1}(0) = \frac{1}{-7} \begin{bmatrix} -3 & -1 \\ -1 & 2 \end{bmatrix}, \quad y(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$y_p(t) = \Phi(t) \int_{t_0}^{t} \Phi^{-1}(s) B(s)\, ds$$

$y' = Ay + B(t)$

$y'' + 3y' = 3\sin t$

* Belirsiz katsayılar, parametre değişimi

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$A = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$,

$\lambda_1 = 2i \rightarrow v_1 = \begin{bmatrix} i \\ -1 \end{bmatrix}$

$\lambda_2 = -2i \rightarrow v_2 = \begin{bmatrix} -i \\ -1 \end{bmatrix}$.

$X' = AX \longrightarrow X(t) = c_1 \begin{bmatrix} i \\ -1 \end{bmatrix} e^{2it} + c_2 \begin{bmatrix} -i \\ -1 \end{bmatrix} e^{-2it}$

$\Phi(t) = \begin{bmatrix} i e^{2it} & -i e^{-2it} \\ -e^{2it} & -e^{-2it} \end{bmatrix}$, $\Phi(0) = \begin{bmatrix} i & -i \\ -1 & -1 \end{bmatrix}$, $\Phi^{-1}(0) = \begin{bmatrix} -\frac{1}{2}i & -\frac{1}{2} \\ \frac{1}{2}i & -\frac{1}{2} \end{bmatrix}$

$e^{At} = \Phi(t)\Phi^{-1}(0)$

$= \begin{bmatrix} i e^{2it} & -i e^{-2it} \\ -e^{2it} & -e^{-2it} \end{bmatrix} \begin{bmatrix} -\frac{1}{2}i & -\frac{1}{2} \\ \frac{1}{2}i & -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2}e^{2it} + \frac{1}{2}e^{-2it} & -\frac{1}{2}ie^{2it} + \frac{1}{2}ie^{-2it} \\ \frac{1}{2}ie^{2it} - \frac{1}{2}ie^{-2it} & \frac{1}{2}e^{2it} + \frac{1}{2}e^{-2it} \end{bmatrix}$

$-ie^{i(2t)} = -i\cos(2t) + \sin(2t)$

$ie^{i(-2t)} = i\cos(2t) + \sin(2t)$

$= \begin{bmatrix} \cos(2t) & \sin(2t) \\ -\sin(2t) & \cos(2t) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cos(2t) + \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \sin(2t)$

$I\cos(2t) + \frac{A}{2} \cdot \sin(2t) = e^{At}$

$\Phi(t) \cdot \Phi^{-1}(0)$

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$X' = \begin{bmatrix} 2 & -2 \\ 2 & -2 \end{bmatrix} X$, $X(0) = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ sistemini üstel matris yöntemiyle çözünüz.

↳ $X(t) = e^{At} \cdot X_0$

$e^{At} = I + At + \frac{A^2t^2}{2!} + \frac{A^3t^3}{3!} + \dots$ $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$A^4 = 0$

$A^2 = \begin{bmatrix} 2 & -2 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} 2 & -2 \\ 2 & -2 \end{bmatrix} = \begin{bmatrix} 4-4 & -4+4 \\ 4-4 & -4+4 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$A^3 = A^2 \cdot A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 2 & -2 \\ 2 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \dots$ $A^n = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $n \neq 0, 1$

$e^{At} = I + At = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 2 & -2 \\ 2 & -2 \end{bmatrix} \cdot t = \begin{bmatrix} 1+2t & -2t \\ 2t & 1-2t \end{bmatrix}$, $X_0 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

$X_t = e^{At} \cdot X_0 = \begin{bmatrix} 1+2t & -2t \\ 2t & 1-2t \end{bmatrix}\begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 1+2t-6t \\ 2t+3-6t \end{bmatrix} = \begin{bmatrix} 1-4t \\ 3-4t \end{bmatrix} = X_t$

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$r^2-4r+3=0 \Rightarrow (r-3)(r-1)=0 \quad r=1,\ r=3$ $\quad c_1e^{x}+c_2e^{3x}$

$y''-4y'+3y=1$, $\boxed{y(0)=0,\ y'(0)=-\frac{1}{3}}$ B.D.P'nin özel çözümünü bulunuz.

$c_1+c_2=0$

$c_1e^{x}+3c_2e^{3x}$

$c_1+3c_2=-\frac{1}{3}$

$\mathcal{L}\{y''-4y'+3y\}=\mathcal{L}\{1\}$ ($\mathcal{L}$ lineerdir) $\Rightarrow \mathcal{L}\{y''\}-4\mathcal{L}\{y'\}+3\mathcal{L}\{y\}=\mathcal{L}\{1\}$

$\left[s^2\mathcal{L}[y(t)]-s\,y(0)-y'(0)\right]-4\left[s\mathcal{L}[y(t)]-y(0)\right]+3\mathcal{L}[y(t)]=\mathcal{L}[1]$

$s^2\mathcal{L}[y(t)]+\frac{1}{3}-4s\mathcal{L}\{y(t)\}+3\mathcal{L}\{y(t)\}=\frac{1}{s}$

$\mathcal{L}\{y(t)\}[s^2-4s+3]=\frac{1}{s}-\frac{1}{3}$

$\mathcal{L}\{y(t)\}=\frac{\frac{1}{s}-\frac{1}{3}}{s^2-4s+3}=\frac{(3-s)}{3s(s-3)(s-1)}=-\frac{1}{3}\cdot\frac{1}{s(s-1)} \Rightarrow \mathcal{L}^{-1}\left[\mathcal{L}\{y(t)\}\right]=y(t)$

$=\mathcal{L}^{-1}\left\{-\frac{1}{3}\cdot\frac{1}{s(s-1)}\right\}$

$\frac{1}{s(s-1)}=\frac{A}{s}+\frac{B}{s-1}$ $\quad A(s-1)+B\cdot s=1$ $\quad A=-1$

$s(A+B)-A=1$ $\quad B=1$

$y(t)=-\frac{1}{3}\mathcal{L}^{-1}\left\{-\frac{1}{s}+\frac{1}{s-1}\right\}=-\frac{1}{3}\left[-1+e^{t}\right]=\boxed{+\frac{1}{3}-\frac{e^{t}}{3}}$

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$$\mathcal{L}^{-1}\left\{\frac{s^2-s+5}{s(s^2+2s+5)}\right\} = ?$$

$$\frac{s^2-s+5}{s(s^2+2s+5)} = \frac{A}{s} + \frac{Bs+C}{s^2+2s+5} \Rightarrow$$

$$A=1,\ B=0,\ C=-3$$

$$\mathcal{L}^{-1}\left\{\frac{s^2-s+5}{s(s^2+2s+5)}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s} - \frac{3}{s^2+2s+5}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - \mathcal{L}^{-1}\left\{\frac{3}{s^2+2s+5}\right\}$$

$$-\frac{3}{2}\,\mathcal{L}^{-1}\left\{\frac{2}{(s+1)^2+2^2}\right\}$$

$$= 1 - \frac{3}{2}\,e^{-t}\cdot\sin(2t)$$

$$\mathcal{L}\{te^{-t} + \sin(3t)\} = \mathcal{L}\{te^{-t}\} + \mathcal{L}\{\sin(3t)\}$$

$$(-1)\cdot\left(\frac{1}{s+1}\right)' + \frac{3}{s^2+3^2} = \frac{1}{(s+1)^2} + \frac{3}{s^2+9}$$

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$g(t) = \begin{cases} e^{2t}, & 0 \le t < 3 \\ 0, & t > 3 \end{cases}$ fonksiyonun Laplace dönüşümü

$$g(t) = e^{2t}\left(1 - u(t-3)\right) = e^{2t} - e^{2t}\cdot u(t-3) = e^{2t} - e^{2\left((t-3)+3\right)}\cdot u(t-3)$$

$$= e^{2t} - e^{6}\cdot e^{2(t-3)}\cdot u(t-3)$$

$$\mathcal{L}\{g(t)\} = \mathcal{L}\{e^{2t}\} - e^{6}\mathcal{L}\{e^{2(t-3)}\cdot u(t-3)\}$$

$$= \frac{1}{s-2} - e^{6}\left(e^{-3s}\cdot\frac{1}{s-2}\right) = \frac{1}{s-2} - \frac{e^{6-3s}}{s-2} = \frac{1-e^{6-3s}}{s-2}$$

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$$\begin{cases} x' = x + 2y \\ y' = x + e^{-t} \\ x(0)=0,\ y(0)=0 \end{cases}$$

$$\mathcal{L}\{x(t)\} = X(s)$$
$$\mathcal{L}\{y(t)\} = Y(s)$$

$$sX(s) = X(s) + 2Y(s)$$
$$sY(s) = X(s) + \frac{1}{s+1}$$

$$X(s)[s-1] - 2Y(s) = 0$$

$$-X(s) + sY(s) = \frac{1}{s+1} \quad / (s-1)$$

$$X(s)[s(s-1)] - 2sY(s) = 0$$

$$-2X(s) + 2sY(s) = \frac{2}{s+1}$$

$$X(s)(s^2 - s - 2) = \frac{2}{s+1} \Rightarrow X(s) = \frac{2}{(s+1)^2(s+2)}$$

$$X(s)[s-1] - 2Y(s) = 0$$

$$-X(s)(s-1) + s(s-1)Y(s) = \frac{s-1}{s+1}$$

$$\Rightarrow Y(s)(s^2 - s - 2) = \frac{s-1}{s+1}$$

$$Y(s) = \frac{s-1}{(s+1)^2(s-2)}$$

$$\frac{2}{(s-2)(s+1)^2} = \frac{A}{s-2} + \frac{B}{(s+1)} + \frac{C}{(s+1)^2} \Rightarrow A = 2/9,\ B = -2/9,\ C = -6/9$$

$$x(t) = \mathcal{L}^{-1}\{X(s)\} = \mathcal{L}^{-1}\left(\frac{2}{9}\cdot\frac{1}{s-2}\right) + \mathcal{L}^{-1}\left(-\frac{2}{9}\cdot\frac{1}{s+1}\right) + \mathcal{L}^{-1}\left(-\frac{6}{9}\cdot\frac{1}{(s+1)^2}\right) = \frac{2}{9}\left[e^{2t} - e^{-t} - 3te^{-t}\right]$$

$$y(t) = \frac{1}{9}\left(e^{2t} - e^{-t} + 6e^{-t}\cdot t\right)$$

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} = \\ = \end{bmatrix}$$

$-y'' + xy' = 0$, $(x-1)$'in kuvvetleri cinsinden kuvvet serisi çözümünü bulunuz

$$y = \sum_{n=0}^{\infty} c_n (x-1)^n \quad , \quad y' = \sum_{n=1}^{\infty} n c_n (x-1)^{n-1} \quad , \quad y'' = \sum_{n=2}^{\infty} n(n-1)(x-1)^{n-2}$$

$$y'' + xy' = \sum_{n=2}^{\infty} n(n-1)(x-1)^{n-2} + x \sum_{n=1}^{\infty} n c_n (x-1)^{n-1} = 0$$

$x = (x-1)+1$

$$\sum_{n=2}^{\infty} c_n n(n-1)(x-1)^{n-2} + \sum_{n=1}^{\infty} n c_n (x-1)^n + \sum_{n=1}^{\infty} n c_n (x-1)^{n-1} = 0$$

$n \to n+2$ $\qquad$ $n \to n+1$

$$\sum_{n=0}^{\infty} (n+2)(n+1)(x-1)^n c_{n+2} + \sum_{n=1}^{\infty} n c_n (x-1)^n + \sum_{n=0}^{\infty} (n+1) c_{n+1} (x-1)^n$$

$2c_2 + \sum_{n=1}^{\infty} (n+2)(n+1) c_{n+2} (x-1)^n$ $\qquad$ $c_1 + \sum_{n=1}^{\infty} (n+1) c_{n+1} (x-1)^n$

$$2c_2 + c_1 + \sum_{n=1}^{\infty} \left[(n+2)(n+1) c_{n+2} + (n+1) c_{n+1} + n c_n \right] (x-1)^n = 0$$

$c_2 = -\frac{1}{2} c_1$

$$c_{n+2} = \frac{-n c_n - (n+1) c_{n+1}}{(n+1)(n+2)}$$

$n=1 \Rightarrow \quad c_3 = \frac{-c_1 - 2c_2}{2 \cdot 3} = 0$

$n=2 \Rightarrow \quad c_4 = \frac{-2c_2 - 3c_3}{3 \cdot 4} = \frac{c_1}{12}$

$n=3 \Rightarrow \quad c_5 = -\frac{c_1}{60}$

$$y(x) = c_0 + c_1(x-1) + c_2(x-1)^2 + c_3(x-1)^3 + \dots$$

$$= c_0 + c_1(x-1) - \frac{1}{2} c_1 (x-1)^2 + \frac{c_1}{12} (x-1)^4 - \frac{c_1}{60} (x-1)^5 + \dots$$

$$= c_0 + c_1 \left[(x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{12}(x-1)^4 - \frac{1}{60}(x-1)^5 + \dots \right]$$

$y_2(x)$

$y_1(x) = 1$

$$y(x) = c_0 \cdot y_1(x) + c_1 y_2(x)$$