TABLE OF LAPLACE TRANSFORM FORMULAS

$$\mathscr{L}[t^n] = \frac{n!}{s^{n+1}} \qquad \mathscr{L}^{-1}\left[\frac{1}{s^n}\right] = \frac{1}{(n-1)!}t^{n-1}$$

$$\mathscr{L}[e^{at}] = \frac{1}{s-a} \qquad \mathscr{L}^{-1}\left[\frac{1}{s-a}\right] = e^{at}$$

$$\mathscr{L}[\sin at] = \frac{a}{s^2+a^2} \qquad \mathscr{L}^{-1}\left[\frac{1}{s^2+a^2}\right] = \frac{1}{a}\sin at$$

$$\mathscr{L}[\cos at] = \frac{s}{s^2+a^2} \qquad \mathscr{L}^{-1}\left[\frac{s}{s^2+a^2}\right] = \cos at$$

First Differentiation Formula

$$\mathscr{L}[D^n x] = s^n\mathscr{L}[x] - s^{n-1}x(0) - s^{n-2}x'(0) - \cdots - x^{(n-1)}(0)$$

$$\mathscr{L}\left[\int_0^t f(u)\,du\right] = \frac{1}{s}\mathscr{L}[f(t)] \qquad \mathscr{L}^{-1}\left[\frac{1}{s}F(s)\right] = \int_0^t \mathscr{L}^{-1}[F(s)]\,du$$

In the following formulas, $F(s) = \mathscr{L}[f(t)]$, so $f(t) = \mathscr{L}^{-1}[F(s)]$.

First Shift Formula

$$\mathscr{L}[e^{at}f(t)] = F(s-a) \qquad \mathscr{L}^{-1}[F(s)] = e^{at}\mathscr{L}^{-1}[F(s+a)]$$

Second Differentiation Formula

$$\mathscr{L}[t^n f(t)] = (-1)^n\frac{d^n}{ds^n}\mathscr{L}[f(t)] \qquad \mathscr{L}^{-1}\left[\frac{d^nF(s)}{ds^n}\right] = (-1)^n t^n f(t)$$

Second Shift Formula

$$\mathscr{L}[u_a(t)g(t)] = e^{-as}\mathscr{L}[g(t+a)] \qquad \mathscr{L}^{-1}[e^{-as}F(s)] = u_a(t)f(t-a)$$

Convolution

$$\mathscr{L}^{-1}[F(s)G(s)] = \mathscr{L}^{-1}[F(s)] * \mathscr{L}^{-1}[G(s)]$$

where

$$(f*g)(t) = \int_0^t f(t-u)g(u)\,du.$$

Convolutions of Trigonometric Functions

$$(\sin\alpha t) * (\cos\alpha t) = \frac{t}{2}\sin\alpha t$$

$$(\sin\alpha t) * (\sin\alpha t) = \frac{1}{2\alpha}\sin\alpha t - \frac{t}{2}\cos\alpha t$$

$$(\cos\alpha t) * (\cos\alpha t) = \frac{1}{2\alpha}\sin\alpha t + \frac{t}{2}\cos\alpha t$$