1) $\left.\begin{array}{r} x + y - z = 0 \\ 2x + 3y + z = c \\ 4x + 7y + az = b \end{array}\right\}$ denklem sistemi a, b ve c nin hangi değerleri için

a) Hiçbir çözümü yoktur b) Tek çözümü vardır c) Sonsuz çözüme sahiptir.

ANAHTARI

① $\left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 2 & 3 & 1 & c \\ 4 & 7 & a & b \end{array}\right] \xrightarrow{r_2 - 2r_1 \to r_2} \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 1 & 3 & c \\ 4 & 7 & a & b \end{array}\right] \xrightarrow{r_3 - 4r_1 \to r_3} \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 1 & 3 & c \\ 0 & 3 & a+4 & b \end{array}\right] \longrightarrow$

$\xrightarrow{r_3 - 3r_2 \to r_3} \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 1 & 3 & c \\ 0 & 0 & a-5 & b-3c \end{array}\right]$

a) $a-5=0$ ve $b-3c \neq 0 \Rightarrow a=5$ ve $b \neq 3c$

Bu durumda çözüm yoktur.

b) $a-5 \neq 0 \Rightarrow a \neq 5$ iken $(a-5).z \neq 0$

Bu durumda tek çözüme sahiptir.

c) $a-5=0$ ve $b-3c=0 \Rightarrow a=5$ ve $b=3c$

Bu durumda sistem sonsuz çözüme sahiptir.

3) A matrisinin tersini elemanter matrislerin çarpımı şeklinde yazarak hesaplayınız.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}.$$

③ $$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix} \xrightarrow{r_3 - r_1 \to r_3} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 0 & -2 & 5 \end{bmatrix} \xrightarrow{r_2 - 2r_1 \to r_2} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -3 \\ 0 & -2 & 5 \end{bmatrix} \xrightarrow{r_1 + r_3 \to r_1} \begin{bmatrix} 1 & 0 & 8 \\ 0 & 1 & -3 \\ 0 & -2 & 5 \end{bmatrix} \rightarrow$$

$$\xrightarrow{r_3 + 2r_2 \to r_3} \begin{bmatrix} 1 & 0 & 8 \\ 0 & 1 & -3 \\ 0 & 0 & -1 \end{bmatrix} \xrightarrow{r_1 + 8r_3 \to r_1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -3 \\ 0 & 0 & -1 \end{bmatrix} \xrightarrow{-r_3 \to r_3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{r_2 + 3r_3 \to r_2} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = B$$

$$\boxed{B = \underbrace{E_7 E_6 E_5 E_4 E_3 . E_2 . E_1}_{A^{-1}} . A}$$

Burada; $E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$, $E_2 = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $E_3 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$E_4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$, $E_5 = \begin{bmatrix} 1 & 0 & 8 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $E_6 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$, $E_7 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$

$$A^{-1} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}} \cdot \underbrace{\begin{bmatrix} 1 & 0 & 8 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}} \cdot \underbrace{\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -3 \\ 0 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 16 & 8 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}} \cdot \underbrace{\begin{bmatrix} 1 & 0 & 1 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}} = \underbrace{\begin{bmatrix} 1 & 16 & 8 \\ 0 & -5 & -3 \\ 0 & -2 & -1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 1 \\ -2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}} = \underbrace{\begin{bmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{bmatrix}}_{A^{-1}}$$

3. $\begin{vmatrix} x+a & b & c & d \\ a & x+b & c & d \\ a & b & x+c & d \\ a & b & c & x+d \end{vmatrix}$ determinantının değerini, determinantın elemanter işlemlerini kullanıp üçgen forma getirerek hesaplayınız. (15 puan)

A^{-1}

$$\begin{vmatrix} x+a & b & c & d \\ a & x+b & c & d \\ a & b & x+c & d \\ a & b & c & x+d \end{vmatrix} \underset{\substack{r_1 \to r_1 - r_4 \\ r_2 \to r_2 - r_4 \\ r_3 \to r_3 - r_4}}{=} \begin{vmatrix} x & 0 & 0 & -x \\ 0 & x & 0 & -x \\ 0 & 0 & x & -x \\ a & b & c & x+d \end{vmatrix}$$

$$\underset{c_4 \to c_4 + c_1}{=} \begin{vmatrix} x & 0 & 0 & 0 \\ 0 & x & 0 & -x \\ 0 & 0 & x & -x \\ a & b & c & (x+d+a) \end{vmatrix} \underset{c_4 \to c_4 + c_2}{=} \begin{vmatrix} x & 0 & 0 & 0 \\ 0 & x & 0 & 0 \\ 0 & 0 & x & -x \\ a & b & c & x+d+a+b \end{vmatrix}$$

$$\underset{c_4 \to c_4 + c_3}{=} \begin{vmatrix} x & 0 & 0 & 0 \\ 0 & x & 0 & 0 \\ 0 & 0 & x & 0 \\ a & b & c & x+a+b+c+d \end{vmatrix} = x^3.(x+a+b+c+d)$$

"Üçgen forma getir, det bul."

(1) i) $\begin{vmatrix} 2 & 1 & 3 \\ 3 & 2 & 1 \\ 2 & 1 & 1 \end{vmatrix} \xrightarrow{-r_1+r_3 \to r_3} \begin{vmatrix} 2 & 1 & 3 \\ 3 & 2 & 1 \\ 0 & 0 & -2 \end{vmatrix} \xrightarrow{\frac{-3r_1}{2}+r_2 \to r_2} \begin{vmatrix} 2 & 1 & 3 \\ 0 & \frac{1}{2} & \frac{-7}{2} \\ 0 & 0 & -2 \end{vmatrix} = -2$