Ad Soyad:
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MAT-201 LİNEER CEBİR VİZE SINAV SORULARI 13/10/2018

1) $\left.\begin{aligned} x+\ y-z&=0\\ 2x+3y+z&=c\\ 4x+7y+az&=b \end{aligned}\right\}$ denklem sistemi a, b ve c nin hangi değerleri için

a) Hiçbir çözümü yoktur b) Tek çözümü vardır c) Sonsuz çözüme sahiptir.

2) $\left.\begin{aligned} 3x-5y+2z&=1\\ x+4y-z&=2\\ -10x+y-2z&=3 \end{aligned}\right\}$ lineer denklem sistemini Cramer yöntemi ile çözünüz.

3) A matrisinin tersini elemanter matrislerin çarpımı şeklinde yazarak **hesaplayınız**.

$$A=\begin{bmatrix} 1 & 2 & 3\\ 2 & 5 & 3\\ 1 & 0 & 8 \end{bmatrix}.$$

4) Aşağıda verilen altkümeler, belirtilen vektör uzaylarının birer alt vektör uzayı mıdır? Araştırınız.

i) $W=\left\{\begin{bmatrix} a+b\\ b+2c\\ a-c \end{bmatrix};\ a,b,c\in\mathbb{R}\right\}\subset\mathbb{R}^3$ ii) $W=\left\{at^2+bt+c;\ a+b=5\right\}\subset P_2$

iii) $W=\left\{\begin{bmatrix} a & b & c\\ 0 & d & m \end{bmatrix};\ d\times m=2\right\}\subset M_{2\times 3}$

5) Aşağıdaki determinantları açmadan, determinantın elemanter satır veya sutün işlemlerini kullanarak

$$\begin{vmatrix} a_1+b_1t & a_2+b_2t & a_3+b_3t\\ a_1t+b_1 & a_2t+b_2 & a_3t+b_3\\ c_1 & c_2 & c_3 \end{vmatrix}=\left(1-t^2\right)\begin{vmatrix} a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3 \end{vmatrix}$$

eşitliğini gösteriniz.

Not: Sınav süresi 120 dakikadır. Sorular eşit puanlıdır. Tam puan almak için Yeterli ve Gerekli açıklamaları yaparak soruları cevaplandırınız.

B A Ş A R I L A R

MAT 201 VİZE CEVAP ANAHTARI

① $\left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 2 & 3 & 1 & c \\ 4 & 7 & a & b \end{array}\right] \xrightarrow{r_2-2r_1\to r_2} \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 1 & 3 & c \\ 4 & 7 & a & b \end{array}\right] \xrightarrow{r_3-4r_1\to r_3} \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 1 & 3 & c \\ 0 & 3 & a+4 & b \end{array}\right] \longrightarrow$

$\xrightarrow{r_3-3r_2\to r_3} \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 1 & 3 & c \\ 0 & 0 & a-5 & b-3c \end{array}\right]$

a) $a-5=0$ ve $b-3c\neq 0 \Rightarrow a=5$ ve $b\neq 3c$

Bu durumda çözüm yoktur.

b) $a-5\neq 0 \Rightarrow a\neq 5$ iken $(a-5).z\neq 0$

Bu durumda tek çözüme sahiptir.

c) $a-5=0$ ve $b-3c=0 \Rightarrow a=5$ ve $b=3c$

Bu durumda sistem sonsuz çözüme sahiptir.

② $A=\begin{bmatrix} 3 & -5 & 2 \\ 1 & 4 & -1 \\ -10 & 1 & -2 \end{bmatrix}$, $b=\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, 2. satıra göre açalım:

$\det A = (-1)^{2+1}.\begin{vmatrix} -5 & 2 \\ 1 & -2 \end{vmatrix} + 4.(-1)^{2+2}.\begin{vmatrix} 3 & 2 \\ -10 & -2 \end{vmatrix} + (-1).(-1)^{2+3}\begin{vmatrix} 3 & - \\ -10 & \end{vmatrix}$

$= -(10-2)+4(-6+20)+(3-50) = 1$

Cramer Yöntemine göre;

$$x_1 = \frac{\begin{vmatrix} 1 & -5 & 2 \\ 2 & 4 & -1 \\ 3 & 1 & -2 \end{vmatrix}}{\det A} = \frac{1.(-1)^{1+1}.\begin{vmatrix} 4 & -1 \\ 1 & -2 \end{vmatrix} + (-5).(-1)^{1+2}.\begin{vmatrix} 2 & -1 \\ 3 & -2 \end{vmatrix} + 2.(-1)^{1+3}.\begin{vmatrix} 2 & 4 \\ 3 & 1 \end{vmatrix}}{1} = -32$$

$$x_2 = \frac{\begin{vmatrix} 3 & 1 & 2 \\ 1 & 2 & -1 \\ -10 & 3 & -2 \end{vmatrix}}{\det A} = \frac{1.(-1)^{1+2}.\begin{vmatrix} 1 & 2 \\ 3 & -2 \end{vmatrix} + 2.(-1)^{2+2}\begin{vmatrix} 3 & 2 \\ -10 & -2 \end{vmatrix} + (-1).(-1)^{2+3}.\begin{vmatrix} 3 & 1 \\ -10 & 3 \end{vmatrix}}{1} = 55$$

$$x_3 = \frac{\begin{vmatrix} 3 & -5 & 1 \\ 1 & 4 & 2 \\ -10 & 1 & 3 \end{vmatrix}}{\det A} = \frac{(-1)^{2+1}\cdot\begin{vmatrix} -5 & 1 \\ 1 & 3 \end{vmatrix} + 4\cdot(-1)^{2+2}\cdot\begin{vmatrix} 3 & 1 \\ -10 & 3 \end{vmatrix} + 2\cdot(-1)^{2+3}\cdot\begin{vmatrix} 3 & -5 \\ -10 & 1 \end{vmatrix}}{1} = 186$$

③ $$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix} \xrightarrow{r_3 - r_1 \to r_3} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 0 & -2 & 5 \end{bmatrix} \xrightarrow{r_2 - 2r_1 \to r_2} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -3 \\ 0 & -2 & 5 \end{bmatrix} \xrightarrow{r_1 + r_3 \to r_1} \begin{bmatrix} 1 & 0 & 8 \\ 0 & 1 & -3 \\ 0 & -2 & 5 \end{bmatrix} \to$$

$$\xrightarrow{r_3 + 2r_2 \to r_3} \begin{bmatrix} 1 & 0 & 8 \\ 0 & 1 & -3 \\ 0 & 0 & -1 \end{bmatrix} \xrightarrow{r_1 + 8r_3 \to r_1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -3 \\ 0 & 0 & -1 \end{bmatrix} \xrightarrow{-r_3 \to r_3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{r_2 + 3r_3 \to r_2} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = B$$

$$\boxed{B = \underbrace{E_7 E_6 E_5 E_4 E_3 . E_2 . E_1}_{A^{-1}} . A}$$

Burada; $E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$, $E_2 = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $E_3 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$E_4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$, $E_5 = \begin{bmatrix} 1 & 0 & 8 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $E_6 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$, $E_7 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$

$$A^{-1} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}} . \underbrace{\begin{bmatrix} 1 & 0 & 8 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}} . \underbrace{\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}} . \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -3 \\ 0 & 0 & -1 \end{bmatrix} . \begin{bmatrix} 1 & 16 & 8 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}} . \underbrace{\begin{bmatrix} 1 & 0 & 1 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}} = \underbrace{\begin{bmatrix} 1 & 16 & 8 \\ 0 & -5 & -3 \\ 0 & -2 & -1 \end{bmatrix} . \begin{bmatrix} 0 & 0 & 1 \\ -2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}} = \underbrace{\begin{bmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{bmatrix}}_{A^{-1}}$$

④ i) $W = \left\{ \begin{bmatrix} a+b \\ b+2c \\ a-c \end{bmatrix} ; a,b,c \in \mathbb{R} \right\} \subset \mathbb{R}^3$, $\forall u,v \in W$, $k \in \mathbb{R}$ için $u + kv \in W$ olmalıdır.

$a=0$, $b=1$, $c=2$ için $\begin{bmatrix} 1 \\ 5 \\ -2 \end{bmatrix} \in W$ old. $W \neq \emptyset$. $u = \begin{bmatrix} a_1+b_1 \\ b_1+2c_1 \\ a_1-c_1 \end{bmatrix}$, $kv = \begin{bmatrix} k(a_2+b_2) \\ k(b_2+2c_2) \\ k(a_2-c_2) \end{bmatrix} \in W$

$u + kv = \begin{bmatrix} a_1+ka_2+b_1+kb_2 \\ b_1+kb_2+2c_1+2kc_2 \\ a_1+ka_2-c_1-kc_2 \end{bmatrix} = \begin{bmatrix} a_3+b_3 \\ b_3+2c_3 \\ a_3-c_3 \end{bmatrix} \in W$ old. W, $\mathbb{R}^3$ için alt uzaydır.

ii) $W = \{at^2+bt+c \ ; \ a+b=5\} \subset P_2$, $\forall p_1, p_2 \in W$ ve $k \in \mathbb{R}$ için $p_1+kp_2 \overset{?}{\in} W$

$$\left.\begin{array}{l} p_1 = a_1t^2+b_1t+c_1, \ (a_1+b_1=5) \\ p_2 = a_2t^2+b_2t+c_2, \ (a_2+b_2=5) \end{array}\right\} \ p_1+kp_2 = t^2(a_1+ka_2)+t(b_1+kb_2)+c_1+kc_2$$

$kp_2 = ka_2t^2+kb_2t+kc_2, (ka_2+kb_2=5)$ burada $a_1+ka_2+b_1+kb_2$

$= 5+5 \neq 10$

O halde W, P_2 için alt uzay değildir.

iii)

$$W = \left\{ \begin{bmatrix} a & b & c \\ 0 & d & m \end{bmatrix} ; d \times m = 2 \right\} \subset M_{2\times3} , \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 1 \end{bmatrix} \in W \text{ old. } W \neq \emptyset .$$

$A = \begin{bmatrix} a_1 & b_1 & c_1 \\ 0 & d_1 & m_1 \end{bmatrix}$, $B = \begin{bmatrix} a_2 & b_2 & c_2 \\ 0 & d_2 & m_2 \end{bmatrix} \in W$ alalım. $\forall k \in \mathbb{R}$ için;

Burada $d_1 \times m_1 = 2$, $d_2 \times m_2 = 2$

$$A+kB = \begin{bmatrix} a_1+ka_2 & b_1+kb_2 & c_1+kc_2 \\ 0 & d_1+kd_2 & m_1+km_2 \end{bmatrix}$$

için $(d_1+kd_2) \times (m_1+km_2) \neq 2$ old. W, $M_{2\times3}$ için alt uzay değildir.

5)

$$\begin{vmatrix} a_1+b_1t & a_2+b_2t & a_3+b_3t \\ a_1t+b_1 & a_2t+b_2 & a_3t+b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \xlongequal{r_2 - tr_1 \to r_2} \begin{vmatrix} a_1+b_1t & a_2+b_2t & a_3+b_3t \\ b_1-b_1t^2 & b_2-b_2t^2 & b_3-b_3t^2 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$\xlongequal{r_2+r_1 \to r_1} \begin{vmatrix} a_1+b_1 & a_2+b_2 & a_3+b_3 \\ b_1(1-t^2) & b_2(1-t^2) & b_3(1-t^2) \\ c_1 & c_2 & c_3 \end{vmatrix} \xlongequal{\frac{-r_2}{(1-t^2)}+r_1 \to r_1} \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1(1-t^2) & b_2(1-t^2) & b_3(1-t^2) \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$\xlongequal{\frac{r_2}{(1-t^2)} \to r_2} (1-t^2) \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$