

Adı Soyadı:

No:

İMZA:

1. (18 p.)	2. (18 p.)	3. (15 p.)	4. (15 p.)	5. (14 p.)	6. (20 p.)	TOPLAM

NOT: Tam puan almak için yeterli açıklama yapılması gerekmektedir.  
Sınav süresi 100 dakikadır. Başarılar.

1.  $\begin{cases} x+2y+z = 2 \\ 2x-2y+3z = 1 \\ x+2y-(a^2-3)z = a \end{cases}$  denklem sistemi veriliyor. Hangi  $a$  değerleri için bu sistemin

- a) Çözümü yoktur b) Tek çözümü vardır c) Sonsuz çözüme sahiptir. (20 puan)

$$A = \left[ \begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 2 & -2 & 3 & 1 \\ 1 & 2 & 3-a^2 & a \end{array} \right] \xrightarrow{r_3 \rightarrow r_3 - r_1} \left[ \begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 2 & -2 & 3 & 1 \\ 0 & 0 & 2-a^2 & a-2 \end{array} \right] \xrightarrow{r_2 \rightarrow r_2 - 2r_1} \left[ \begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & -4 & 1 & -3 \\ 0 & 0 & 2-a^2 & a-2 \end{array} \right]$$

a) Çözüm Yok  $\Rightarrow 2-a^2=0$  ve  $a-2 \neq 0$  olmali  $\Rightarrow a^2=2$  ve  $a \neq 2 \Rightarrow a=\pm\sqrt{2}$

b) Tek Çözüm  $\Rightarrow 2-a^2 \neq 0$  olmali  $\Rightarrow a^2 \neq 2 \Rightarrow a \neq \pm\sqrt{2}$

c) Sonsuz Çözüm  $\Rightarrow 2-a^2=0$  ve  $a-2=0 \Rightarrow a^2=2$  ve  $a=2$

Böyle bir durum

$\Rightarrow a=\pm\sqrt{2}$  ve  $a=2 \Rightarrow$  söz konusu değildir.

2.  $A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 2 \\ 3 & 2 & 0 \end{bmatrix}$  matrisinin tersini elemanter matrislerin çarpımı şeklinde ifade edip hesaplayınız. (15 puan)

$$A = \left[ \begin{array}{ccc} 1 & 1 & 2 \\ 2 & 1 & 2 \\ 3 & 2 & 0 \end{array} \right] \xrightarrow{E_1: r_2 \rightarrow r_2 - 2r_1} \left[ \begin{array}{ccc} 1 & 1 & 2 \\ 0 & -1 & -2 \\ 3 & 2 & 0 \end{array} \right] \xrightarrow{E_2: r_3 \rightarrow r_3 - 3r_1} \left[ \begin{array}{ccc} 1 & 1 & 2 \\ 0 & -1 & -2 \\ 0 & -1 & -6 \end{array} \right] \xrightarrow{E_3: r_1 \rightarrow r_2 + r_1} \left[ \begin{array}{ccc} 1 & 0 & 0 \\ 0 & -1 & -2 \\ 0 & -1 & -6 \end{array} \right] \xrightarrow{E_4: r_3 \rightarrow r_3 - r_2} \left[ \begin{array}{ccc} 1 & 0 & 0 \\ 0 & -1 & -2 \\ 0 & 0 & -4 \end{array} \right]$$

$$I = \left[ \begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right] \xleftarrow{E_7: r_2 \rightarrow r_2 - 2r_3} \left[ \begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{array} \right] \xleftarrow{E_6: r_2 \rightarrow -r_2} \left[ \begin{array}{ccc} 1 & 0 & 0 \\ 0 & -1 & -2 \\ 0 & 0 & 1 \end{array} \right] \xleftarrow{E_5: r_3 \rightarrow \frac{r_3}{-4}} \left[ \begin{array}{ccc} 1 & 0 & 0 \\ 0 & -1 & -2 \\ 0 & 0 & -4 \end{array} \right]$$

$A^{-1}$

$$I = E_7 \cdot E_6 \cdot E_5 \cdot E_4 \cdot E_3 \cdot E_2 \cdot E_1 \cdot A$$

$$\Rightarrow E_1 = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

$$E_3 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, E_4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}, E_5 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix}, E_6 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, E_7 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -\frac{3}{2} & \frac{1}{2} \\ 0 & \frac{1}{4} & \frac{-1}{4} \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ -2 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 \\ \frac{3}{2} & -\frac{3}{2} & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{4} & \frac{-1}{4} \end{bmatrix}$$

$E_7 \cdot E_6$

$E_5 \cdot E_4$

$E_3 \cdot E_2$

$E_1$

$E_7 \cdot E_6 \cdot E_5 \cdot E_4$

$E_3 \cdot E_2 \cdot E_1$

$\overbrace{\qquad\qquad\qquad}^{A^{-1}}$

$$3. \left| \begin{array}{cccc} x+a & b & c & d \\ a & x+b & c & d \\ a & b & x+c & d \\ a & b & c & x+d \end{array} \right|$$

determinantının değerini, determinantın elemanter işlemlerini kullanıp

üçgen forma getirerek hesaplayınız. (15 puan)

$$\left| \begin{array}{cccc} x+a & b & c & d \\ a & x+b & c & d \\ a & b & x+c & d \\ a & b & c & x+d \end{array} \right| \begin{array}{l} r_1 \rightarrow r_1 - r_4 \\ r_2 \rightarrow r_2 - r_4 \\ r_3 \rightarrow r_3 - r_4 \end{array} \left| \begin{array}{cccc} x & 0 & 0 & -x \\ 0 & x & 0 & -x \\ 0 & 0 & x & -x \\ a & b & c & x+d \end{array} \right|$$

$$\begin{array}{l} c_4 \rightarrow c_4 + c_1 \\ \hline \end{array} \left| \begin{array}{cccc} x & 0 & 0 & 0 \\ 0 & x & 0 & -x \\ 0 & 0 & x & -x \\ a & b & c & (x+d+a) \end{array} \right| \begin{array}{l} c_4 \rightarrow c_4 + c_2 \\ \hline \end{array} \left| \begin{array}{cccc} x & 0 & 0 & 0 \\ 0 & x & 0 & 0 \\ 0 & 0 & x & -x \\ a & b & c & x+d+a+b \end{array} \right|$$

$$\begin{array}{l} c_4 \rightarrow c_4 + c_1 \\ \hline \end{array} \left| \begin{array}{cccc} x & 0 & 0 & 0 \\ 0 & x & 0 & 0 \\ 0 & 0 & x & 0 \\ a & b & c & x+a+b+c+d \end{array} \right| = x^3 \cdot (x+a+b+c+d)$$

4.  $A^{-1} = \frac{\text{Adj } A}{\det A}$

$$\text{Adj } A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

matrisi veriliyor. Buna göre  $A$  matrisinin tersini, Adjoint matris yardımıyla hesaplayınız. (20 puan)

$$A_{11} = (-1)^2 \cdot \begin{vmatrix} 5 & 3 \\ 0 & 8 \end{vmatrix} = 40$$

$$A_{21} = (-1)^3 \cdot \begin{vmatrix} 2 & 3 \\ 0 & 8 \end{vmatrix} = -16$$

$$A_{31} = (-1)^4 \cdot \begin{vmatrix} 2 & 3 \\ 5 & 3 \end{vmatrix} = -$$

$$A_{12} = (-1)^3 \cdot \begin{vmatrix} 2 & 3 \\ 1 & 8 \end{vmatrix} = -13$$

$$A_{22} = (-1)^4 \cdot \begin{vmatrix} 1 & 3 \\ 1 & 8 \end{vmatrix} = 5$$

$$A_{32} = (-1)^5 \cdot \begin{vmatrix} 1 & 3 \\ 2 & 3 \end{vmatrix} = 3$$

$$A_{13} = (-1)^4 \cdot \begin{vmatrix} 2 & 3 \\ 1 & 0 \end{vmatrix} = -5$$

$$A_{23} = (-1)^5 \cdot \begin{vmatrix} 1 & 2 \\ 1 & 0 \end{vmatrix} = 2$$

$$A_{33} = (-1)^6 \cdot \begin{vmatrix} 1 & 2 \\ 2 & 5 \end{vmatrix} = 1$$

$$\Rightarrow \text{Adj } A = \begin{bmatrix} 40 & -16 & -9 \\ -13 & 5 & 3 \\ -5 & 2 & 1 \end{bmatrix}$$

$$3. \text{sütuna göre sırasıktır. } \det A = 0 \quad A_{31} + a_{32} A_{32} + a_{33} A_{33} = 1(-9) + 8,1 = -1$$

$$\Rightarrow A^{-1} = \frac{\text{Adj } A}{\det A} = \begin{bmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{bmatrix} \text{ olurken bulunur.}$$

$$5. \left. \begin{array}{l} x^2 + y^2 + z^2 = 6 \\ x^2 - y^2 + 2z^2 = 2 \\ 2x^2 + y^2 - z^2 = 3 \end{array} \right\} \text{ denklem sistemi veriliyor. Buna göre } x, y \text{ ve } z \text{ değerlerini Cramer metodu ile bulunuz. (20 puan)}$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ 2 & 1 & -1 \end{bmatrix}, b = \begin{bmatrix} 6 \\ 2 \\ 3 \end{bmatrix}$$

1. sütuna göre sırasıktır.

$$\det A = A_{11} + A_{21} + A_{31}$$

$$= (-1)^2 \cdot \begin{vmatrix} -1 & 2 \\ 1 & -1 \end{vmatrix} + (-1)^3 \cdot \begin{vmatrix} 1 & 2 \\ 1 & -1 \end{vmatrix} + (-1)^4 \cdot \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix}$$

Cramer Metodunu göre;

$$x^2 = \frac{\begin{vmatrix} 6 & 1 & 1 \\ 2 & -1 & 2 \\ 3 & 1 & -1 \end{vmatrix}}{\det A} = \frac{3 \cdot (-1)^4 \cdot \begin{vmatrix} 1 & 1 \\ -1 & 2 \end{vmatrix} + 1 \cdot (-1)^5 \cdot \begin{vmatrix} 6 & 1 \\ 3 & -1 \end{vmatrix} + 1 \cdot (-1)^6 \cdot \begin{vmatrix} 6 & 1 \\ 2 & -1 \end{vmatrix}}{7} = \frac{7}{7} = 1$$

$$= (1-2) - (-1-2) + (1+2) = 7$$

$$y^2 = \frac{\begin{vmatrix} 1 & 6 & 1 \\ 1 & 2 & 2 \\ 2 & 3 & -1 \end{vmatrix}}{\det A} = \frac{2 \cdot (-1)^4 \cdot \begin{vmatrix} 6 & 1 \\ 2 & 2 \end{vmatrix} + 3 \cdot (-1)^5 \cdot \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} + (-1) \cdot (-1)^6 \cdot \begin{vmatrix} 1 & 6 \\ 1 & 2 \end{vmatrix}}{7} = \frac{21}{7} = 3$$

$$z^2 = \frac{\begin{vmatrix} 1 & 1 & 6 \\ 1 & -1 & 2 \\ 2 & 1 & 3 \end{vmatrix}}{\det A} = \frac{2 \cdot (-1)^4 \cdot \begin{vmatrix} 1 & 6 \\ -1 & 2 \end{vmatrix} + (-1)^5 \cdot \begin{vmatrix} 1 & 6 \\ 1 & 2 \end{vmatrix} + 3 \cdot (-1)^6 \cdot \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix}}{7} = \frac{14}{7} = 2$$

$$\begin{aligned} x &= \mp 1 & \Rightarrow G \cdot K = \{(1, \sqrt{3}, \sqrt{2}), (1, \sqrt{3}, -\sqrt{2}), (1, -\sqrt{3}, \sqrt{2}), (1, -\sqrt{3}, -\sqrt{2}), \\ y &= \mp \sqrt{3} & (-1, \sqrt{3}, \sqrt{2}), (-1, \sqrt{3}, -\sqrt{2}), (-1, -\sqrt{3}, \sqrt{2}), (-1, -\sqrt{3}, -\sqrt{2})\} \\ z &= \mp \sqrt{2} \end{aligned}$$

6.  $V = \mathbb{R} \times \mathbb{R} = \{(x, y) : x, y \in \mathbb{R}\}$  olmak üzere  $V'$  de toplama ve (skaler ile) çarpma işlemleri sırasıyla

$$(u_1, v_1) \oplus (v_1, v_2) = (u_1 + v_1 + 1, u_2 + v_2 + 1)$$

ve  $k \in \mathbb{R}$  için

$$k \odot (u_1, v_1) = (ku_1, kv_1)$$

olarak tanımlanıyor. Buna göre  $(V, \oplus, \odot)$  bir vektör uzayı mıdır? (10 puan)

$(V, \oplus, \odot)$  vektör uzayı değildir, çünkü;

$\forall k \in \mathbb{R}, \forall (u_1, u_2), (v_1, v_2) \in V$  için

$$\underbrace{k \odot ((u_1, u_2) \oplus (v_1, v_2))}_{k \odot (u_1 + v_1 + 1, u_2 + v_2 + 1)} \neq (k \odot (u_1, u_2)) \oplus (k \odot (v_1, v_2))$$

$$(ku_1, ku_2) \oplus (kv_1, kv_2)$$

$$= (k(u_1 + v_1 + 1), k(u_2 + v_2 + 1)) = (ku_1 + kv_1 + 1, ku_2 + kv_2 + 1)$$

$$= (ku_1 + kv_1 + k, ku_2 + kv_2 + k)$$

Özellikini sağlamadı.