30.10.2019

19/20 GÜZ MAT 201 CEVAP ANAHTARI

①

$$\left[\begin{array}{ccc|c} 2 & 3 & 1 & 1 \\ 4 & 5 & 3 & 3 \\ -2 & 3 & 5 & 5 \end{array}\right] \xrightarrow[r_1+r_3\to r_3]{-2r_1+r_2\to r_2} \left[\begin{array}{ccc|c} 2 & 3 & 1 & 1 \\ 0 & -1 & 1 & 1 \\ 0 & 6 & 6 & 6 \end{array}\right] \xrightarrow[6r_2+r_3\to r_3]{3r_2+r_1\to r_1} \left[\begin{array}{ccc|c} 2 & 0 & 4 & 4 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & 12 & 12 \end{array}\right] \xrightarrow[\frac{r_3}{12}\to r_3]{-r_2\to r_2,\ \frac{r_1}{2}\to r_1}$$

$$\to \left[\begin{array}{ccc|c} 1 & 0 & 2 & 2 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right] \xrightarrow[r_3+r_2\to r_2]{-2r_3+r_1\to r_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right] \Rightarrow \begin{array}{l} x_1=0 \\ x_2=0 \\ x_3=1 \end{array} \text{ 'dir.}$$

②

$$\begin{vmatrix} 2 & 3 & 1 & 0 \\ 1 & 2 & 0 & 2 \\ -3 & 2 & 1 & 5 \\ 1 & 3 & 2 & 1 \end{vmatrix} \overset{r_2\leftrightarrow r_1}{=} (-1)\begin{vmatrix} 1 & 2 & 0 & 2 \\ 2 & 3 & 1 & 0 \\ -3 & 2 & 1 & 5 \\ 1 & 3 & 2 & 1 \end{vmatrix} \underset{\substack{3r_1+r_3\to r_3 \\ -r_1+r_4\to r_4}}{\overset{-2r_1+r_2\to r_2}{=}} (-1)\begin{vmatrix} 1 & 2 & 0 & 2 \\ 0 & -1 & 1 & -4 \\ 0 & 8 & 1 & 11 \\ 0 & 1 & 2 & -1 \end{vmatrix} \overset{-r_2\to r_2}{=}$$

$$= \begin{vmatrix} 1 & 2 & 0 & 2 \\ 0 & 1 & -1 & 4 \\ 0 & 8 & 1 & 11 \\ 0 & 1 & 2 & -1 \end{vmatrix} \underset{-r_2+r_4\to r_4}{\overset{-8r_2+r_3\to r_3}{=}} \begin{vmatrix} 1 & 2 & 0 & 2 \\ 0 & 1 & -1 & 4 \\ 0 & 0 & 9 & -21 \\ 0 & 0 & 3 & -5 \end{vmatrix} \overset{\frac{r_3}{-3}+r_4\to r_4}{=} \begin{vmatrix} 1 & 2 & 0 & 2 \\ 0 & 1 & -1 & 4 \\ 0 & 0 & 9 & -21 \\ 0 & 0 & 0 & 2 \end{vmatrix} = 9.2 = 18$$

③

$$A = \begin{bmatrix} 3 & -5 & 2 \\ 1 & 4 & -1 \\ -10 & 1 & -2 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

1. satıra göre;

$$\det A = (-1)^{1+1}.3.\begin{vmatrix} 4 & -1 \\ 1 & -2 \end{vmatrix} + (-1)^{1+2}.(-5).\begin{vmatrix} 1 & -1 \\ -10 & -2 \end{vmatrix} + (-1)^{1+3}.2.\begin{vmatrix} 1 & 4 \\ -10 & 1 \end{vmatrix} = 1$$

$$x_1 = \frac{\begin{vmatrix} 1 & -5 & 2 \\ 2 & 4 & -1 \\ 3 & 1 & -2 \end{vmatrix}}{\det A} \overset{\text{1.satır açılım}}{=} \frac{(-1)^{1+1}.\begin{vmatrix} 4 & -1 \\ 1 & -2 \end{vmatrix} + (-5)(-1)^{1+2}.\begin{vmatrix} 2 & -1 \\ 3 & -2 \end{vmatrix} + 2(-1)^{1+3}.\begin{vmatrix} 2 & 4 \\ 3 & 1 \end{vmatrix}}{1} = -32$$

$$x_2 = \frac{\begin{vmatrix} 3 & 1 & 2 \\ 1 & 2 & -1 \\ -10 & 3 & -2 \end{vmatrix}}{\det A} \overset{\text{1.satır açılım}}{=} \frac{3.(-1)^{1+1}.\begin{vmatrix} 2 & -1 \\ 3 & -2 \end{vmatrix} + (-1)^{1+2}.\begin{vmatrix} 1 & -1 \\ -10 & -2 \end{vmatrix} + 2(-1)^{1+3}.\begin{vmatrix} 1 & 2 \\ -10 & 3 \end{vmatrix}}{1} = 55$$

$$x_3 = \frac{\begin{vmatrix} 3 & -5 & 1 \\ 1 & 4 & 2 \\ -10 & 1 & 3 \end{vmatrix}}{\det A} \overset{\text{1.satır açılım}}{=} \frac{3(-1)^{1+1}.\begin{vmatrix} 4 & 2 \\ 1 & 3 \end{vmatrix} + (-5)(-1)^{1+2}.\begin{vmatrix} 1 & 2 \\ -10 & 3 \end{vmatrix} + (-1)^{1+3}.\begin{vmatrix} 1 & 4 \\ -10 & 1 \end{vmatrix}}{1} = 186$$

4) i) $A=\begin{bmatrix}1&0&1\\2&-1&1\\1&2&1\end{bmatrix}$, $A^{-1}=\dfrac{\text{Adj } A}{\det A}$, $\det A = \overset{\text{1. satır açılım}}{1.(-1)^{1+1}}.\begin{vmatrix}-1&1\\2&1\end{vmatrix}+(-1)^{1+3}.\begin{vmatrix}2&-1\\1&2\end{vmatrix}=2$

$A_{11}=(-1)^{1+1}.\begin{vmatrix}-1&1\\2&1\end{vmatrix}=-3$ $\quad A_{21}=(-1)^{2+1}.\begin{vmatrix}0&1\\2&1\end{vmatrix}=+2$ $\quad A_{31}=(-1)^{3+1}.\begin{vmatrix}0&1\\-1&1\end{vmatrix}=1$

$A_{12}=(-1)^{1+2}.\begin{vmatrix}2&1\\1&1\end{vmatrix}=-1$ $\quad A_{22}=(-1)^{2+2}.\begin{vmatrix}1&1\\1&1\end{vmatrix}=0$ $\quad A_{32}=(-1)^{3+2}.\begin{vmatrix}1&1\\2&1\end{vmatrix}=1$

$A_{13}=(-1)^{1+3}.\begin{vmatrix}2&-1\\1&2\end{vmatrix}=5$ $\quad A_{23}=(-1)^{2+3}.\begin{vmatrix}1&0\\1&2\end{vmatrix}=-2$ $\quad A_{33}=(-1)^{3+3}.\begin{vmatrix}1&0\\2&-1\end{vmatrix}=-1$

$$\text{Adj } A=\begin{bmatrix}A_{11}&A_{21}&A_{31}\\A_{12}&A_{22}&A_{32}\\A_{13}&A_{23}&A_{33}\end{bmatrix}=\begin{bmatrix}-3&2&1\\-1&0&1\\5&-2&-1\end{bmatrix} \Rightarrow A^{-1}=\begin{bmatrix}\frac{-3}{2}&1&\frac{1}{2}\\\frac{-1}{2}&0&\frac{1}{2}\\\frac{5}{2}&-1&\frac{-1}{2}\end{bmatrix} \text{ dir.}$$

i) a) $V=\{at^2+bt+c \; : \; a+b=5,\; c=2a\}\subset P_2$

V'den keyfi iki eleman alalım:

$p_1.p_2\in V \Rightarrow p_1=a_1t^2+b_1t+c_1 \quad (a_1+b_1=5,\; c_1=2a_1)$
$p_2=a_2t^2+b_2t+c_2 \quad (a_2+b_2=5,\; c_2=2a_2)$

$p_1+p_2=(a_1+a_2)t^2+(b_1+b_2)t+(c_1+c_2) \overset{?}{\in} V$

$a_1+a_2+b_1+b_2=(a_1+b_1)+(a_2+b_2)=10\neq 5$ old. $p_1+p_2\notin V$ V alt uzay değildir

b) $W=\left\{\begin{bmatrix}-a&b\\c&-d\end{bmatrix} : a-b=c-d\right\}\subset M_2$

W'dan keyfi iki eleman alalım: $A,B\in W$ ve $k\in\mathbb{R}$ olsun.

$A=\begin{bmatrix}-a_1&b_1\\c_1&-d_1\end{bmatrix}$, $B=\begin{bmatrix}-a_2&b_2\\c_2&-d_2\end{bmatrix}$ ve $\begin{matrix}a_1-b_1=c_1-d_1\\a_2-b_2=c_2-d_2\end{matrix}$ için $\begin{matrix}A+B\overset{?}{\in}W\\k.A\overset{?}{\in}W\end{matrix}$

$A+B=\begin{bmatrix}-(a_1+a_2)&b_1+b_2\\c_1+c_2&-(d_1+d_2)\end{bmatrix}$ ve

$(a_1+a_2)-(b_1+b_2)=a_1-b_1+a_2-b_2=c_1-d_1+c_2-d_2=(c_1+c_2)-(d_1+d_2)$ ✓

$\Rightarrow A+B\in W$ ✓.

$$k.A = \begin{bmatrix} -ka_1 & kb_1 \\ kc_1 & -kd_1 \end{bmatrix} \quad \text{ve} \quad ka_1 - kb_1 = k(a_1-b_1) = k(c_1-d_1) = kc_1 - kd_1 \quad ✓$$

$\Rightarrow k.A \in W$ ✓

O halde W, M_2 uzayının altuzaydır.

5) i) $\begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix} = c_1 \begin{bmatrix} 2 & 1 \\ 1 & -3 \end{bmatrix} + c_2 \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} + c_3 \begin{bmatrix} 1 & -3 \\ -1 & 2 \end{bmatrix} + c_4 \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$ sağlayan $c_1, c_2, c_3, c_4 \in \mathbb{R}$ old. göstermeliyiz.

$$\begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 2c_1+c_2+c_3+4c_4 & c_1+2c_2-3c_3+3c_4 \\ c_1-c_3+2c_4 & -3c_1+3c_2+2c_3+c_4 \end{bmatrix} \Rightarrow \begin{aligned} 2c_1+c_2+c_3+4c_4 &= 2 \\ c_1+2c_2-3c_3+3c_4 &= 3 \\ c_1-c_3+2c_4 &= 4 \\ -3c_1+3c_2+2c_3+c_4 &= 1 \end{aligned}$$

$$\Rightarrow \left[\begin{array}{cccc|c} 2 & 1 & 1 & 4 & 2 \\ 1 & 2 & -3 & 3 & 3 \\ 1 & 0 & -1 & 2 & 4 \\ -3 & 3 & 2 & 1 & 1 \end{array}\right] \xrightarrow[3r_3+r_4 \to r_4]{r_1 \leftrightarrow r_2} \left[\begin{array}{cccc|c} 1 & 2 & -3 & 3 & 3 \\ 2 & 1 & 1 & 4 & 2 \\ 1 & 0 & -1 & 2 & 4 \\ 0 & 3 & -1 & 7 & 13 \end{array}\right] \xrightarrow[-r_1+r_3 \to r_3]{-2r_1+r_2 \to r_2} \left[\begin{array}{cccc|c} 1 & 2 & -3 & 3 & 3 \\ 0 & -3 & 7 & -2 & -4 \\ 0 & -2 & 2 & -1 & 1 \\ 0 & 3 & -1 & 7 & 13 \end{array}\right] \longrightarrow$$

$$\xrightarrow[2r_3+r_2 \to r_2]{r_1+r_3 \to r_1} \left[\begin{array}{cccc|c} 1 & 0 & -1 & 2 & 4 \\ 0 & 1 & 3 & 0 & -6 \\ 0 & -2 & 2 & -1 & 1 \\ 0 & 3 & -1 & 7 & 13 \end{array}\right] \xrightarrow[-3r_2+r_4 \to r_4]{2r_2+r_3 \to r_3} \left[\begin{array}{cccc|c} 1 & 0 & -1 & 2 & 4 \\ 0 & 1 & 3 & 0 & -6 \\ 0 & 0 & 8 & -1 & -11 \\ 0 & 0 & -10 & 7 & 31 \end{array}\right] \xrightarrow{r_3+r_4 \to r_3} \left[\begin{array}{cccc|c} 1 & 0 & -1 & 2 & 4 \\ 0 & 1 & 3 & 0 & -6 \\ 0 & 0 & -2 & 6 & 20 \\ 0 & 0 & -10 & 7 & 31 \end{array}\right]$$

$$\xrightarrow{\frac{r_3}{2} \to r_3} \left[\begin{array}{cccc|c} 1 & 0 & -1 & 2 & 4 \\ 0 & 1 & 3 & 0 & -6 \\ 0 & 0 & 1 & -3 & -10 \\ 0 & 0 & -10 & 7 & 31 \end{array}\right] \xrightarrow[-3r_3+r_2 \to r_2]{r_1+r_3 \to r_1 \quad 10r_3+r_4 \to r_4} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & -6 \\ 0 & 1 & 0 & +9 & 24 \\ 0 & 0 & 1 & -3 & -10 \\ 0 & 0 & 0 & -23 & -69 \end{array}\right] \xrightarrow{\frac{-r_4}{23} \to r_4} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & -6 \\ 0 & 1 & 0 & +9 & 24 \\ 0 & 0 & 1 & -3 & -10 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right]$$

$$\xrightarrow[r_4+r_3 \to r_3]{+r_4 \to r_1 \quad r_4+r_2 \to r_2} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -3 \\ 0 & 1 & 0 & 0 & -3 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right] \Rightarrow \begin{aligned} c_1 &= -3 \\ c_2 &= -3 \\ c_3 &= -1 \\ c_4 &= 3 \end{aligned} \quad \text{ve} \quad c_1, c_2, c_3, c_4 \in \mathbb{R} \text{ old.}$$

$\begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$ elemanı bahsedilen kümenin elemanlarının gerdiği uzaya aittir.

ii) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ -3 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ m \end{bmatrix} \right\}$ kümesinin $\mathbb{R}^3$ uzayını germesi için ;

o.s. $c_1, c_2, c_3 \in \mathbb{R}$ mevcut olmalıdır.

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ -3 \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} 2 \\ -1 \\ m \end{bmatrix} = \begin{bmatrix} c_1 - c_2 + 2c_3 \\ 2c_1 - 3c_2 - c_3 \\ c_1 + c_2 + mc_3 \end{bmatrix} \Rightarrow \begin{matrix} c_1 - c_2 + 2c_3 = a \\ 2c_1 - 3c_2 - c_3 = b \\ c_1 + c_2 + mc_3 = c \end{matrix}$$ için ;

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & a \\ 2 & -3 & -1 & b \\ 1 & 1 & m & c \end{array}\right] \xrightarrow[-r_1 + r_3 \to r_3]{-2r_1 + r_2 \to r_2} \left[\begin{array}{ccc|c} 1 & -1 & 2 & a \\ 0 & -1 & -5 & b-2a \\ 0 & 2 & m-2 & c-a \end{array}\right] \xrightarrow{2r_2 + r_3 \to r_3} \left[\begin{array}{ccc|c} 1 & -1 & 2 & a \\ 0 & -1 & -5 & b-2a \\ 0 & 0 & m-12 & 2b-5a+c \end{array}\right]$$

Bu sistemin çözümünün olabilmesi için ;

$m-12 \neq 0 \Rightarrow m \neq 12$ iken ve $\forall a,b,c \in \mathbb{R}$ için bu sistem her zaman

çözüme sahiptir.