Math 1920 – Basic Integration Formulas

Remember that you can always check/verify integration problems/formulas by simply differentiating your answer.

Basic Properties/Formulas/Rules

1. $\int kf(x)\,dx = k\int f(x)\,dx$

2. $\int f(x) \pm g(x)\,dx = \int f(x)\,dx \pm \int g(x)\,dx$

Polynomial/Rational Integrals

3. $\int 0\,dx = C$

4. $\int k\,dx = kx + C$

5. $\int x^n\,dx = \dfrac{x^{n+1}}{n+1} + C, \quad n \neq -1$

6. $\int x^{-1}\,dx = \int \dfrac{dx}{x} = \ln|x| + C$

Trig Integrals

7. $\int \sin x\,dx = -\cos x + C$

8. $\int \cos x\,dx = \sin x + C$

9. $\int \sec^2 x\,dx = \tan x + C$

10. $\int \csc^2 x\,dx = -\cot x + C$

11. $\int \sec x \tan x\,dx = \sec x + C$

12. $\int \csc x \cot x\,dx = -\csc x + C$

Exponential Integrals

13. $\int e^x\,dx = e^x + C$

14. $\int a^x\,dx = \dfrac{a^x}{\ln a} + C$

Hyperbolic Integrals

15. $\int \sinh x\,dx = \cosh x + C$

16. $\int \cosh x\,dx = \sinh x + C$

17. $\int \operatorname{sech}^2 x\,dx = \tanh x + C$

18. $\int \operatorname{csch}^2 x\,dx = -\coth x + C$

19. $\int \operatorname{sech} x \tanh x\,dx = -\operatorname{sech} x + C$

20. $\int \operatorname{csch} x \coth x\,dx = -\operatorname{csch} x + C$

Inverse Trig/Hyperbolic Integrals

21. $\int \dfrac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\dfrac{x}{a}\right) + C$

22. $\int \dfrac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left(x + \sqrt{x^2 \pm a^2}\right) + C$

23. $\int \dfrac{dx}{a^2 + x^2} = \dfrac{1}{a}\tan^{-1}\left(\dfrac{x}{a}\right) + C$

24. $\int \dfrac{dx}{a^2 - x^2} = \dfrac{1}{2a}\ln\left|\dfrac{a+x}{a-x}\right| + C$

25. $\int \dfrac{dx}{x\sqrt{x^2 - a^2}} = \dfrac{1}{a}\sec^{-1}\left(\dfrac{x}{a}\right) + C$

26. $\int \dfrac{dx}{x\sqrt{a^2 \pm x^2}} = -\dfrac{1}{a}\ln\left(\dfrac{a + \sqrt{a^2 \pm x^2}}{|x|}\right) + C$

Note: The inverse **trig** integrals (left column) will look more familiar if you let $a = 1$. However, letting a be an arbitrary constant we can derive the formulas above, which are a great deal more general (hence much more useful).

Note: The inverse **hyperbolic** integrals (right column) may not be immediately recognizable because the right-hand sides are written using the logarithmic definitions of the inverse hyperbolic functions. This is customarily done because it allows you to write all five integrals using only the three equations above. As with the inverse trig integrals, they will look more familiar if you let $a = 1$ (and look at the inverse hyperbolic definitions). Again, letting a be arbitrary yields formulas that are more general.